Enhanced 3-D OCDMA Code Family using Asymmetric Run Length Constraints

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***Abstract*—This paper suggests an enhanced performance of the 3-D optical code division multiple access (OCDMA) codes, a space/wavelength/time spreading family of codes. The initial codes are in the format wavelength hopping/time sequence (WH/TS), selected according to their performance requirements and the TS sequence is constructed to achieve a linear space-time complexity. The asymmetric run length constraints are introduced in that regard, such that the positive bit positions align with the encoder/decoder frequency spacing pattern, yielding a 3-D WH/WS/TS. The selected 2-D OCDMA codes are one-coincidence frequency hopping codes (OCFHC) and optical orthogonal codes (OOC). As a time sequence code, the OOC code length is extended with a code rate of 0.04. The complexity and the bit error rate (BER) are herein given and compared with previous work. The results of the performance show not only an improvement in the number of simultaneous users due to the code length extension, but better correlation properties and hence a better signal-to-noise ratio.**

***Index Terms*—Optical orthogonal code (OOC), optical code division multiple access (OCDMA), difference of position (DoP), neighbor difference, asymmetric run length constraints.**

I. Introduction

The optical communication in future must provide platform that enables access network convergence. This implies transparency to all existing and future access networks. With this requirement, particularly in terms of transmission, any *wavelength division multiplexing* (WDM) based technology will be costly, even if constructed with a 3-D coding based solution. Such a 3-D solution, space/wavelength/time sequence was proposed in [1] using an hybrid WDM-OCDMA, prime codes and *wavelength array gratings* (WAG) as encoder. First, this solution is costly in terms of wavelengths available; second the prime sequences used are of less performance than the *optical orthogonal code* (OOC); and finally the 3-D S/W/T $O(w^3)$ complexity of these code constructions does not allow the electrical decoder to match its optical counterpart. To remedy these gaps, we suggest a complete *optical code division multiple access* (OCDMA) system using *Bragg gratings* (BG) encoder/decoder. In [2], it is shown that the BG chain is a suitable for 2-D *wavelength/time sequence* (WH/TS) encoding, with [3] proving that *one-coincidence frequency hopping code/optical orthogonal code* (OCFHC/OOC) is by far a better selected sequence due to the OOC characteristics.

In this paper, we construct the TS OOC taking into account the bit "1" positions to yield a linear function in the space-time plane with the aim to reduce complexity. This complexity reduction is further exploited by introducing the zero run length constraints called asymmetric run length constraints in compliance with the encoder/decoder wavelength pattern to achieve a *wavelength spacing/time sequence* (WS/TS). The final sequence is then WH/WS/TS. Provided that the fiber's portion required for the gratings is not of infinite length, the WS/TS sequence is constructed such that the code rate is around 0.04, which means an n-bits/m-bits mapping coding scheme for OOC that limits the code length sequence and consequently the Bragg gratings chain length. Furthermore, in an OCDMA system, the transmission is impaired by the *multiple access interference* (MAI) which noise contribution variance is obtained from the more accurate analysis as conducted in [4]. The impact of the new TS structure constructed using the neighbor difference is assessed through the resulting impairment variance and a comparison is thereof made with [1] results.

For the above purpose, the organization of this paper is as follows. We will start in Section II with the theory behind the selected code, followed by presenting the neighbor difference code construction in Section III. In Section IV after having determined the code length bounds and the subsequent extension, we introduce the zero length constraints, the performance of this operation investigated in Section V and compared with the 3-D OCDMA family from literature.

II. Background

Optical orthogonal code, denoted $(n, w, \lambda_a, \lambda_c)$-OOC, is a family of (0,1) sequences representing each a code word of length n, Hamming weight w, auto- and cross-correlation indexes λ_a and λ_c, respectively, satisfying the following properties [5]:

- Auto-correlation: $\sum_{i=0}^{n-1} X_i X_{i+\tau} \leq \lambda_a$
- Cross-correlation: $\sum_{i=0}^{n-1} X_i Y_{i+\tau} \leq \lambda_c$

with the subscript τ, a positive integer, taken in modulo n, for any $X = (x_0, x_1, \ldots, x_{n-1})$, $Y = (y_0, y_1, \ldots, y_{n-1})$ code vectors of C.

Throughout this paper we only consider the OOC with $\lambda_a = \lambda_c = \lambda = 1$. This code is used as the *time spreading* (TS) code for user signature sequences, a line code as well as symmetric error correcting code, though we emphasize only its line characteristics. As a user signature code, it is worthy to construct optimal OOC in terms of cardinality which is linked to the code length n through $\lfloor \frac{n-1}{w(w-1)} \rfloor$ [6] from Johnson's bounds [7]. Already, this optimal solution was reached in [8] for $w = 3$ if and only if $n \neq 6t+2$, with $t \cong 2$ or $3 \pmod 4$, but a lot is still to be done for $w \geq 4$. In [9] and [10], solutions for $(n, 4, 1)$ and $(9n, 4, 1)$ were suggested based on incomplete difference matrix and prime congruent. In [11], Colbourn demonstrated a wider extension of code length. To achieve the extension of the code we are presenting herein, a construction solution called neighbor difference introduced by [12], is revisited as well as the code length bounds with respect to the congruence class requirements are given. The lower bounds of the code length is obtained from the difference family given by Chunk and Khumar [13] derived from Wilson $(n, w, 1)$-OOC construction based on Galois Field $GF(n)$ [14], where n is not necessarily prime, $n \equiv 1 \mod (w(w-1))$, while the upper bounds come from the optimal Johnson bounds [7]. An OCFHC of length L and Q frequencies available denoted by $(L, Q, H_{\max})$ is a $(0, 1)$ sequence, with 0 if no frequency is present, and 1 otherwise, satisfying the Hamming correlation condition $H_{\max} = 1$ and optimal cardinality $\frac{Q(Q-1)}{L}$ [3].

III. Neighbor Difference

We suggest the neighbor difference to construct the OOC as it allows the manipulation of the sequence to fit a required pattern. The solution is the partition of the difference of position (DoP), which takes into account bit "1" position in the code vector. The way this code is represented can be found in [7], [18] and it is well analyzed using a combinatorial method which more so ever yields optimal OOC. The analysis of different combinatorial methods can be found in [15], [16], [17] and [5] for cyclic packing, and [19] for difference cyclic packing led the neighbor difference correlation as the following subsection defines.

1) Definition: The i-th neighbor is the set $X_1 = \{\delta_{i,1}, \delta_{i,2}, \ldots, \delta_{i,w-1}\}$ where $\delta_{i,j}$ represents the difference in position between the $(j+i)$-th symbol "1" and the j-th symbol "1" [12].

Example 1 $C = \{1101000\}$ is a $(7, 3, 1)$ code with one code word, $C_1 = \{1, 2\}$ is the first neighbor set and $C_2 = \{3\}$ is the second neighbor set.

The neighbor difference on an (n, w, λ)-OOC can be considered in this case as a family F of $w-1$ subsets of neighbor positions $X_i = \{\delta_{ij} : 1 \leq j \leq w-i\}$ and $1 \leq i \leq w-1$, such that the difference of position based on the i-th neighbor $\delta_{i,j} = |b_{i,j+1} - b_{i,j}|$ is represented in at most $\lambda = 1$ times. As a result, one can define the following correlation properties:

- Auto-correlation: for each $b_x \in X$, a code vector of the neighbor difference, any integer $\delta \neq 0$ can be represented as the difference $\delta_{ij} = |b_{x,j+i} - b_{x,j}| : j \neq i,\ i, j = 0, 1, \ldots, w-1, b \in X, b \neq 0$ in at most λ blocks.
- Cross-correlation: for each $b_x \in X$ and $b_y \in Y$, two code vectors, any integer $\delta \neq 0$ can be represented as the difference $\delta_{x,ij} = |b_{x,j+i} - b_{x,j}|$ and $\delta_{y,ij} = |b_{y,j+i} - b_{y,j}| : j \neq i,\ i, j = 0, 1, \ldots, w-1, b \in X, b \neq 0$ in at most λ blocks.

2) Neighbor difference derivation: The neighbor difference can be derived from the cyclic difference packing (CDP), the from where the difference correlation definition results in the matrix given by

$$\begin{bmatrix} 0 & \delta_{0,1} & \delta_{0,2} & \cdots & \delta_{0,w-1} \\ \delta_{1,0} & 0 & \delta_{1,2} & \cdots & \delta_{2,w-1} \\ \delta_{2,0} & \delta_{2,1} & 0 & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \delta_{w-2,w-1} \\ \delta_{w-1,0} & \delta_{w-1,1} & \delta_{w-1,2} & \cdots & 0 \end{bmatrix} \tag{1}$$

From this matrix, the upper triangular matrix elements define the neighbor difference elements, that can be reduced to the following neighbor difference matrix where the diagonal is made of first neighbors:

$$\begin{bmatrix} \delta_{0,1} & \delta_{0,2} & \cdots & \delta_{0,w-1} \\ 0 & \delta_{1,2} & \cdots & \delta_{1,w-1} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \delta_{w-2,w-1} \end{bmatrix} \tag{2}$$

The construction of the code is reduced in finding the elements of the matrix (2) with $\delta_{i,j} \neq 0$ yielding $\frac{w(w-1)}{2}$ operations. One realizes that the diagonal elements are enough to position the positive bits which corresponds to the gratings on the Bragg gratings chain. These elements can be put into notation set $X = \{0, \delta_{0,1}, \delta_{1,2}, \ldots, \delta_{w-2,w-1}\}$ which can be converted into a vector by padding the "1" to express the code word $X = [1, \delta_{0,1} = 1, (\delta_{1,2} + \delta_{0,1}) = 1, \ldots, (\delta_{w-2,w-1} + \cdots + \delta_{0,1}) = 1]$ (other positions are "0"). This reduces the number of operations to $2(w-1)$.

A. Extension of $n \cong 1 \mod (w(w-1))$

The extension comes with the requirements of having optimal OOC in terms of users on the one hand, and mitigating the line impairments on the other hand. Therefore, this extension is done with respect to the full orbit requirement of [10], the difference family of [11] and the bounds of the code length.

1) Bounds of the Code Length Requirements:

a) Upper Bound Code Length: The upper bound on the code length is derived from the cardinality of the code. The cardinality of an $(n, w, 1)$-OOC is given by Johnson's bound $|C| \leq \lfloor \frac{n-1}{w(w-1)} \rfloor$ [19] and the equality means that the OOC is optimal.

Consider l as the largest integer resulting from the ratio $\frac{n-1}{w(w-1)}$, then one can write $n_{\max} \geq w(w-1)l + n'$ where $1 \leq n' \leq w(w-1)\epsilon : 0 < \epsilon < 1$, therefore $1 \leq n' < w(w-1)$.

b) Lower Bound Code Length: The lower bound is generated by the neighbor positions of bits "1" taken at their minimum values as follows.

Define the i-th and the k-th neighbor of the j-th bit "1" by $1 \leq i \neq k \leq w-1$. If for any $i \neq k$ neighbor of any j-th bit "1" position the neighbor differences are such that $\delta_{k,j} = \delta_{k,j-1} + 1$ and $\delta_{i,j} = \delta_{i,j-1} + 1$, and if furthermore the maximum run of bits "0" after the last bit "1" in the sequence $r \pmod n$ is the minimum neighbor difference with respect to the same arithmetic progression available, the first neighbors

$\delta_{1,j}$ containing the entire information of the sequence, then the minimum code length is given by:

$$n_{\min} = \sum_{j=1}^{w-1} \delta_{1,j} + r \pmod{n}.$$

Hence the following proposition.

Proposition 1 *For any starter block B_i of the family $F = \{B_1, B_2, \dots, B_l\}$ of an $(n, w, 1)$-OOC, if $\delta_{k,J} = \delta_{k,j} + 1$, for any $i \neq k$, then the code length n is bounded as follows: $\sum_{j=1}^{w-1} \delta_{1,j} + r \pmod{n} \leq n \leq w(w-1)l + n'$ where n' is such that $1 \leq n' < w(w-1)l$, l is the number of blocks called code words, and the lower bound is necessarily $\sum_{j=1}^{w-1} \delta_{1,j} + r \pmod{n} \cong 1 \mod (w(w-1))$.*

Proof: Assume $X = x_0 x_1 \cdots x_{n-1}$ a code vector of an OOC where the following differences are observed

$$\begin{aligned} x_1 - x_0 &= r_0 \\ x_2 - x_1 &= r_0 + 1 \\ x_3 - x_2 &= r_0 + 2 \\ \vdots \quad &\quad \vdots \\ \sum_{i \neq j = 0}^{3} (x_i - x_j) &= 3r_0 + 3 \end{aligned} \tag{3}$$

This can be extended at $(n-1)$-th order by writing:

$$\sum_{i,j} (x_i - x_j) = (n-1)r_0 + (n-1) = w(w-1).$$

Since each bit "1" has $(w-1)$ differences and if $r_0 = 0$, then one can write $w(w-1) = n - 1$. Therefore $n \cong 1 \mod (w(w-1))$ is the lower bound code length. ∎

2) Full Orbit Requirements: From the theoretical points, for a pair (Z_n, B) if for any block $B_i = \{b_{i1}, \dots, b_{iw}\}$, $1 \leq i \leq l$, the w-set of the following block $B_i + 1 = \{b_{i1+1}, \dots, b_{iw+1}\} \pmod{n}$, $1 \leq i \leq l$ is also a block. The orbit containing B_i is therefore the set of blocks $\{B_i + j \pmod{v} : j = 0, 1, \dots, v-1\}$. Such a block is noted $CB(n, w, 1)$ [11]. The cycle length is the number of blocks in the orbit. If the cycle length is equal to the code length, then the blocks of the orbit are of full orbit, otherwise they are short. In this construction, we are interested only in blocks of full orbit.

3) Zero Run Length Constraints: The extension of the code length adds another requirement, the one of having a unique solution in constructing the code. The sequence resulting from these bounds and requirements can be obtained with the introduction of the minimum run length of bit "0" r_0 and the maximum run length of its "0" R_0. The pair (r_0, R_0) is called the bits "0" run length constraints or asymmetric run length constraints. It can be given in a certain pattern that defines the congruence class as in the following proposition.

Proposition 2 *For any block B of an $(n, w, 1)$-OOC, if any code satisfies an arithmetic sequence property $S_0 = 1$ and $S_i = S_{i-1} + 1$, where S_0 is the first term of the sequence and S_i the final term, $i = w$ the number of terms. If $r_0 = S_0 + 1$ and $R_0 = S_i$, then (r_0, R_0) represents the minimum and maximum run length of bits "0", B is unique and the difference leave is $L = \Phi$.*

From this congruence class $n \cong 1 \mod (w(w-1))$, $w = 3$, $r_0 = 1$ and $R_0 = 3$, $m \cong 3 \mod (w(w-1))$, we can now construct the $(n, w, 1)$-OOC using the neighbor difference method.

IV. Zero Run Length Construction of $(n, w, 1)$-OOC

A. Some Considerations

The construction takes into account the following considerations:

- Identify the Hamming weight of the code, w.
- Each element of the matrix (2) defined by δ_{ij} and its complementary $\delta_{ji} = n - \delta_{ij}$ belong to the same block, therefore constitutes one element of the block.
- Introduce the run lengths of "0" (r_0, R_0) in the sequence, keeping in mind that $r_0 \neq 0$ and there is always bit "1" at position 0.
- The pair (r_0, R_0) represents the minimum and maximum zero length constraints which according to Proposition 2 $r_0 = 1$, and $R_0 = R_i$ yields the minimum congruence class $m \cong 3 \mod (w(w-1))$, the difference family adopted here because of a code rate of 0.04.

B. Code Construction: n-bits/m-bits Mapping

We are interested in a class $n = w(w-1) + 1$ and $w \geq 4$, for a block B, there exists $x, y, z, \dots t \in [1, \frac{n-1}{2}]$ representing the values of the δ_{ij} such that $x \neq y \neq z \neq \cdots \neq t$ and $\bar{x} = n - x$, $\bar{y} = n - y$, $\bar{z} = n - z$, ..., $\bar{t} = n - t \notin B$ their complementary elements, the correlation properties and cyclic block operation impose, for any element $x, y \in B$ with $x \neq y$, $\bar{x}$ and $\bar{y} \notin B$, that is, for any set of differences elements $\Delta_B = \{x, y, z, \dots, t, n-t, \dots, n-z, n-y, n-x\}$, that:

- If $1 < x, y \leq \frac{n-1}{2}$, then for any $z, t \in \Delta_B - \{1\}$, $z, t \in B$ if and only if $z \neq x \neq y \neq t$ and $z \neq x + [2(\frac{n-1}{2} - x) + 1]$; similarly, $t \neq x \neq y \neq z$ and $t \neq y + [2(\frac{n-1}{2} - y) + 1]$.
- If $1 < x, y \geq \frac{n-1}{2}$, then for any $z, t \in \Delta_B - \{1\}$, $z, t \in B$ if and only if $z \neq x \neq y \neq t$ and $z \neq x - [2(x - \frac{n+1}{2}) + 1]$; similarly, $t \neq x \neq y \neq z$ and $t \neq y - [2(y - \frac{n+1}{2}) + 1]$.

V. Construction of $(n, w, 1)$-OOC

The following steps are required to construct the code.

- Identify the Hamming weight of the code, w.
- Each element of the matrix (3) defined by δ_{ij} and its complementary $\delta_{ji} = n - \delta_{ij}$ belong to the same block therefore, constitutes one element of the block.
- Introduce the run lengths of "0" (r_0, R_0) in the sequence keeping in mind that $r_0 \neq 0$ and there is always bit "1" at position 0.
- The pair (r_0, R_0) represents the minimum and maximum zero length constraints which according to Proposition 2 $r_0 = 1$, and $R_{0i} = R_{0i-1} + 2$ yields the minimum

congruence class $n = 3 \cong \mod (w(w-1))$, our difference family.

- The starter block has position 0 of the vector filled with bit "1" and the two neighbor differences (r_0+1) and (R_0+1) selected together with their complementary in modulo n operation $(n-(r_0+1))$ and $(n-(R_0+1))$ respectively.
- Determine the remaining $\frac{w(w-1)}{2}-2$ elements of matrix (2). The block is reduced to the diagonal elements of that matrix called the first neighbors.

Example 2 Let us construct an $(n,3,1)$-OOC. The construction of this code is given here within the bounds of its length: $2\delta_{i,0}+1 \leq n \leq w(w-1)l+n'$. The minimum code length is $2\times 3+1=7$ and the maximum is $6l+5$ according to the bounds. Introducing the run length constraints yields $n=9$. The run length are 1 and 3, hence the neighbor differences and their complementaries $(2,7),(4,5)$. Only the neighbor difference $(3,6)$ pair remains where the first neighbor is 3. Hence the following sequence: 1010010000, which corresponds to the block $B=\{0,2,7\}$ and it is unique.

In the same manner, we can construct for a given code length n and Hamming weight w an $(n,w,1)$-OOC and achieve the line code, as Table I illustrates some of the constructions obtained.

VI. Comparison $n \cong 1 \mod (w(w-1))$ and $n \cong 3 \mod (w(w-1))$

The performance analysis of this code extension is meant for MAI mean and variance analysis.

A. Multiple Access Interference

The MAI is evaluated based on correlations as in [4], [20], since there are lags with zero correlation. It is analyzed for (Q,N), the number of frequencies and time pulses spreading user signals. The k-th modulating signal of user data:

$$s_k = \sqrt{2p}A(\omega_q,n)a_k(t-\tau_k)b_k(t-\tau_k)\cos(\omega_q t+\phi_k).$$

$A(\omega_q,n)a_k(t-\tau_k)$ is the OCFHC/OOC spreading sequence where $a_k(t)=\sum_{l=0}^{N-1} a_l^k p(t-lT_c)$ and

$$A(\omega_q,n)=\begin{cases}1, & \text{if } \omega_q \text{ is present at } \tau_k,\\ 0, & \text{otherwise,}\end{cases}$$

represent the OOC time spreading sequence represented by a rectangular pulse and OCFHC sequence, $b_k(t)$ is the data sequence also assume to be $b_k(t)=\sum_{j=0}^{N-1} b_j^k p(t-jT)$ where $T=NT_c$ and b_j^k is the j-th data of the k-th user, (a_l^k) and (b_j^k) being Bernoulli random variables.

At the receiver input the K users signals are such that $r(t)=\sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{q=1}^{Q}\sqrt{2P}A(\omega_q,n)b_j^k p(t-\tau_k)a_l^k(t-\tau_k)\cos(w\omega_c t-\tau_k)$.

The non-coherent receiver listening to user 1 will output:

$$\begin{aligned} y_1 &= \int_0^{T_c} s_1(t)r_1(t)dt \\ &= I_1+\sqrt{2P}\sum_{n=1}^{N-1}\sum_{q=1}^{Q-1} A(\omega_q,n)b_j^k p(t-\tau_k)a_l^k p(t-\tau_k). \end{aligned}$$

The quantity I_1 is the user 1 decision value while the second term represents the multiple access interference:

$$I_{\text{mai}}=\sqrt{2P}\sum_{n=1}^{N-1}\sum_{q=1}^{Q-1} A(\omega_q,n)b_j^k p(t-\tau_k)a_l^k p(t-\tau_k).$$

Therefore, user 1 will experience in average from any k users the following interference:

$$I_{\text{mai}}=\sqrt{2P}\int_0^{T_c}\sum_{k=1}^{K-1} b_j^k a_l^k p(t)a_l^k p(t-\tau_k)dt.$$

Following the same approximation as in [4] and [20], we obtain the following random MAI

$$I_{\text{mai}}=P_b[X_k+Y_k(1-S)+P_k(1-S)+Q_kS],$$

which with the extension yields

$$I_{\text{mai}}=P_b[Y_k(1-S)+P_k(1-S)+Q_kS].$$

Note that X_k,Y_k,P_k,Q_k are interpreted as follows: X_k and Y_k represent the cross-correlation of the spreading sequence coefficients $\{a_k^j\}$; P_k and Q_k are the correlation coefficients at the end and at the beginning (after one cycle) of the sequence positions. Without loss of generality, $S=S_k$ is the random time correlation of the spreading rectangular pulse probability function, considered as uniformly distributed over $[0,T_c)$ and so is $(1-S)$.

Since the MAI noise process is Bernoulli distributed with mean $\mathrm{E}[I_{\text{mai}}]$ and variance $\mathrm{Var}[I_{\text{mai}}]$, it can be approximated as a Gaussian process using the central limit theorem with the density probability function $f(i_{\text{mai}})$ for BER evaluation:

$$f(i_{\text{mai}})=\frac{1}{\sqrt{2\pi\mathrm{Var}[I_{\text{mai}}]}}\exp\left(-\frac{i_{\text{mai}}^2}{2}\right)$$

where

$$i_{\text{mai}}=\frac{I_{\text{mai}}-\mathrm{E}[I_{\text{mai}}]}{\sqrt{\mathrm{Var}[I_{\text{mai}}]}}.$$

TABLE I
Construction of (r_0,R_0) $n\cong 3 \mod (w(w-1))$ OOC

w	n	(r_0,R_0)	Sequence
3	9	$(1,3)$	$\{0,2,5\}$
4	15	$(1,5)$	$\{0,2,5,9\}$
5	23	$(1,7)$	$\{0,2,5,9,17\}$
6	33	$(1,9)$	$\{0,2,5,9,17,23\}$
7	45	$(1,11)$	$\{0,3,7,9,17,22,33,\}$
8	59	$(1,13)$	$\{0,3,5,9,20,27,37,45\}$

VII. Numerical Results

To carry out this simulation, we considered the system model in Fig. 1 and the Gaussian approximation of both the WDM-OCDMA, using prime codes as prescribed by [1], and the MAI above. We chose the sequence that suits the code rate of 0.04 in which the n-bits/m-bits mapping is 7-bits/9-bits for the OOC while the OCFHC remains with the same characteristics. This length extension is investigated through the correlation properties. Both 3-D WDM-OCDMA prime codes and 3-D OCDMA OCFHC/OOC are investigated with the same number of users. The results are given in Figs. 2 and 3.

A. Results Analysis

The results of our construction method as Table I suggests and as Fig. 2 shows, are all code with good correlation properties from the difference family. The construction method shows a reduction of complexity from $O(w^3)$ of the prime sequences to the $O(w^2)$ which is further reduced to $O(w)$, allowing the electrical correlation/decorrelation operations at the detection to match the BG speed. Furthermore, with the introduction of zero run length constraints, one experiences a pulsive like correlations, proof of correlation reduction, as confirmed in Fig. 2. In Fig. 3, it is shown that the overall BER of our approach 3-D ODMA OCFHC/OOC is better than the previous results based on 3-D WDM-OCDMA prime codes though the latter shows a big gap for users less than 100, number ($n = 100$). This solution will be costly to have 100 gratings in a BG chain. Considering this result of [1] however, the observed gap can be bridged easily with the use of the embedded asymmetric error correction capability of the OOC if necessary.

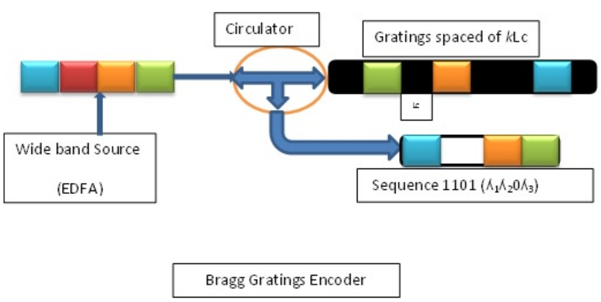

Fig. 1. System model: Bragg Gratings of FFH/OOC scenario.

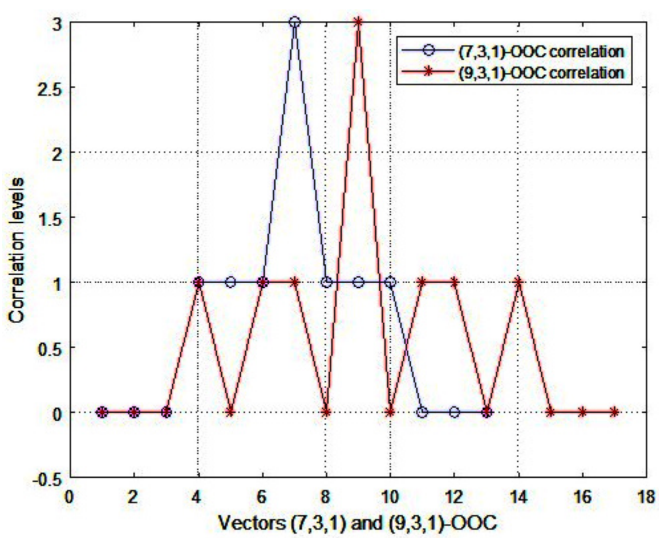

Fig. 2. Auto-correlation: {0,1,3} of an (7,3,1)-OOC vs. {0,2,5} of an (9,3,1)-OOC

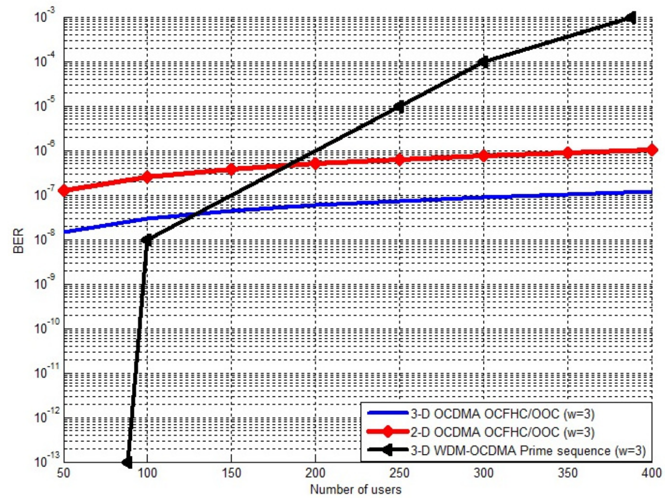

Fig. 3. BER 3-D OCFHC/OOC vs. prime codes

VIII. Conclusion

In this paper, we have reduced the construction complexity of the OOC/OCFHC to $O(w)$ by linearizing the plane (S, T) space-time that transforms the TS sequence into WS/TS, yielding a 3-D WH/WS/TS, compared to $O(w^3)$ of the 3-D S/W/T of [1]. This construction based on 0.04 code rate leads to a better BER due to the reduction of the correlations brought in by the introduction of zero run length constraints, hence an improve number of simultaneous users. The method to correct the asymmetric errors that could occur during transmission, as stated earlier, is left for further studies.

Acknowledgment

This work is based on research supported in part by the National Research Foundation of South Africa (Grant number 109543). The authors would also like to thank B. Twala, previously with the Institute for Intelligent Systems, University of Johannesburg, and C. Mbhowa and S. Motala with the Postgraduate School, University of Johannesburg, for their financial support.

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